### Lab 8-2: Network Flow Lab

**For each of the following problems:**

1. Draw a flow network that represents the problem. Explain how the flow network represents the problem. Including why does the maximum flow obtained represent a solution to the problem; Satisfies the contraints etc.
2. Explain how to tell if there is no solution to the problem.
3. Assuming there is a solution, explain how determine the specific information that specifies the solution. For example, in the dining problem, at which tables are the members of the family seated.

### Problem 1: Dining problem Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to find a seating arrangement that meets this objective (or prove that no such arrangement exists) by using a maximum flow problem. You should draw a picture of the network flow problem using the notation below. Then explain how running the Ford-Fulkerson shortest augmenting path algorithm will enable you to find a solution or determine if no solution exists.

### Assume that the dinner contingent has p families and that the ith family has members and that q tables are available and the jth table has a seating capacity of .

### Problem 2: Committee Representation

Suppose there are five committees A through E each a subset of a set of six persons { a,b,c,d,e,f }. The committees are: A={ b, e}, B={ b, d, e }, C={ a, c, d, e, f }, D={ b, d, e }, E={ b, e }. Is it possible to select a representative from each committee so that no person represents more than 1 committee? If so, what would be an acceptable assignment of persons to represent the committees? You do not need to submit an answer to this specific problem, it is only here to help you think concretely about the general problem.

The general problem of n subsets of some set of m objects and for each subset find a representative object where an object is only allowed to represent one subset. Specify finding the solution to this problem using a flow network. You must use the following notation. Let A be the set of n objects { | i = 1..n }. Assume there are k subsets labeled **⊆ A** j = 1..k.

### Problem 3: Capacity Errors

In a particular flow network G=(V,E,U) whose edges have integer capacities {} , suppose that we have already found the maximum flow f = from vertex 1 (source) to vertex n (the sink).

1. Suppose that the capacity of one of the edges in G is incorrect and should be + 1. Determine if this changes the maximum flow and, if so, also determine the new flow (both in O(|V|+|E|) time.)
2. Suppose that the capacity of one of the edges in G is incorrect and should be ꟷ 1. Determine if this changes the maximum flow and, if so, also determine the new flow (both in O(|V|+|E|) time.)

We could redo the calculation from scratch to see if it makes a difference but there is a faster way.

(For part b., first determine if the value of the optimal flow will stay the same. This will tell you on what edges the flow will need to change to keep the value the same. If you cannot keep the value the same, then determine on what edges the flows need to change for the new max flow.)